

# Instantaneous Online Monitoring of Unmanned Aerial Vehicles without Baseline Signals

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## ABSTRACT

A structural health monitoring problem is often cast in the context of a statistical pattern recognition paradigm, where damage is inferred by comparing test signals with previously recorded baseline signals. However, operational and environmental variations of a system after the collection of the baseline signals can often mask signal changes caused by damage when the statistical pattern comparison is performed. To address this issue for continuous online monitoring, a damage detection technique, which does not rely on any past baseline signals, is proposed to assess damage in composite structures such as wings of an unmanned combat aerial vehicle Predator. A time reversal concept of modern acoustics has been adapted to guided-wave propagation to improve the detectability of local defects in composite structures. It is demonstrated that the original input waveform could be successfully reconstructed in a composite plate through the enhanced time reversal method. However, this time reversibility of Lamb waves is violated when wave distortion due to wave scattering is caused by a defect along a wave propagation path. Examining the deviation of the reconstructed signal from the known initial input signal allows instantaneous identification of damage without requiring the baseline signal for comparison.

## INTRODUCTION

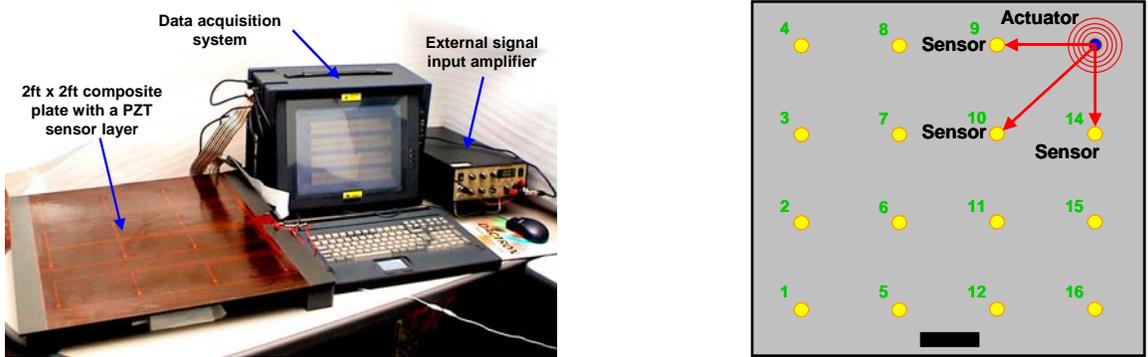
A structural health monitoring problem can often be cast in the context of a statistical pattern recognition paradigm, in which a damage state of the system is inferred by comparing test data measured at a questionable state of the system with baseline data obtained from the initial condition of the system [15]. It has been reported that in-service structures experience large variation of their dynamic characteristics due to their continuously changing operational and environmental conditions [14]. When a damage state of the system is inferred from the pattern comparison of the baseline and test data, it is critical to differentiate the signal changes due to damage from those caused by the undesired operational and environmental variation of the system. This procedure is referred to as data normalization [14]. In this paper, the issue of data normalization is addressed by developing an instantaneous damage detection system that does not require any past baseline signals. By removing the dependency on the prior baseline data, the proposed damage detection system becomes less vulnerable to operational and environmental variation that might occur throughout the lifespan of the system. In stead of comparing the baseline and test signals, a damage-sensitive feature is extracted by applying a known local excitation and comparing the known input with the response signal. This process is based on the concept of time reversal acoustics. Then, a statistical damage classifier is constructed based on a consecutive outlier analysis to identify the location and area of delamination without relying on any knowledge of prior data.

This paper is organized as follows. The experimental setup for detecting delamination in a composite plate is described. Then, the time reversal process used for extracting a damage-sensitive feature is introduced, and a statistical damage classifier is developed based on the concept of the consecutive outlier analysis. The experimental results are presented, and this paper concludes with discussions and summary.

## EXPERIMENTAL SETUP

The overall test configuration of this study is shown in Figure 1 (a). The test setup consists of a composite plate with a surface mounted sensor layer, a personal computer with a built-in data acquisition system, and an external

signal amplifier. The dimension of the composite plates is 60.96 cm x 60.96 cm x 0.6350 cm (24 in x 24 in x 1/4 in). The layout of this composite laminate contains 48 plies stacked according to the sequence [6(0/45/-45/90)]<sub>s</sub>, consisting of Toray T300 Graphite fibers and a 934 Epoxy matrix. A commercially available thin film with embedded Lead Zirconate Titanate (PZT) sensors was mounted on one surface of the composite plate as shown in Figure 1 (b). A total of 16 PZT patches were used as both sensors and actuators to form an “active” local sensing system. Because the PZTs produce an electrical charge when deformed, the PZT patches can be used as dynamic strain gauges. Conversely, the same PZT patches can also be used as actuators, because elastic waves are produced when an electrical field is applied to the patches [18]. In this study, one PZT patch was designated as an actuator, exerting a predefined waveform into the structure. Then, the adjacent PZTs became strain sensors and measure the response signals. This actuator-sensor sensing scheme is graphically shown in Figure 1 (b). This process of the Lamb wave propagation was repeated for different combinations of actuator-sensor pairs. A total of 66 different path combinations were investigated in this study. Actual delamination was seeded to the composite plate by shooting a 185 gram steel projectile into the composite plate. Cables were attached to one side of the plate so that the plate could hang from the test frame in a free-free condition. Several impact tests were repeated varying the impact speed of the steel projectile around 31 m/s to 46 m/s. The data collection using the active sensing system was performed before and after the impact test.



(a) Testing configuration (b) A layout of the PZT sensors/actuators

Figure 1: An active sensing system for detecting delamination on a composite plate

**EXPERIMENTAL SETUP**

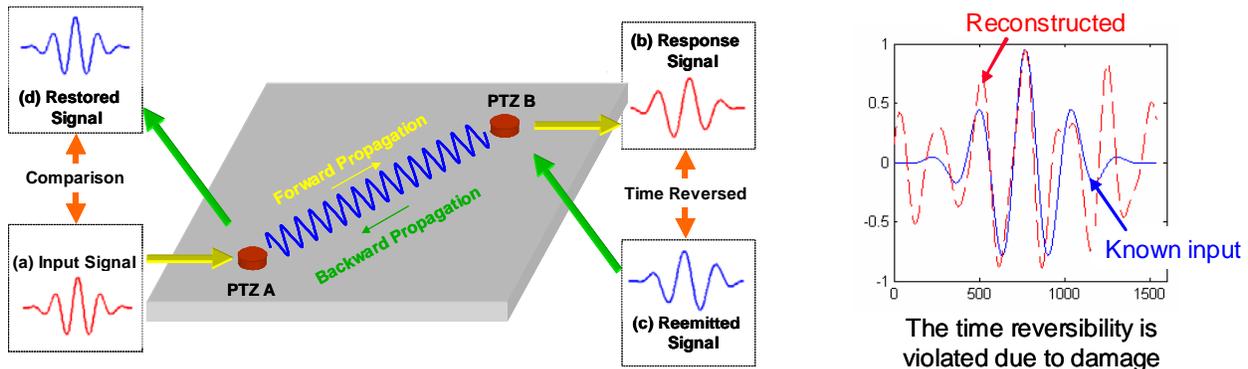
The origin of the time reversal method traces back to the time reversal acoustics [6]. According to the time reversal concept, an input signal can be reconstructed at an excitation point (point A) if an output signal recorded at another point (point B) is reemitted to the original source point (point A) after being reversed in a time domain as shown in Figure 2 (a). This process is referred to as the time reversibility of waves. This time reversibility is based on the spatial reciprocity and time-reversal invariance of linear wave equations [5]. Damage detection using the time reversal process is based on the premise that if there is any nonlinear defect along the wave propagation path, the time reversibility breaks down. By examining the deviation of the reconstructed signal from the known input signal as shown in Figure 1(b), certain types of damage can be identified without requiring any past baseline signals. This time reversibility of body waves has found applications in lithotripsy, ultrasonic brain surgery, nondestructive evaluation, and acoustic communications [7]. Unlike body waves, the propagation of Lamb waves is, however, complicated due to two unique features: dispersion and multimode [20], which has limited the use of Lamb waves for damage detection applications [9]. To alleviate the multimode and dispersion issues of Lamb waves for the time reversal process, a combination of a specific narrowband input waveform design and a multi-resolution signal processing is employed so that the time reversibility of Lamb waves could be preserved within an acceptable tolerance in the presence of background noise [12].

Once the time reversibility of Lamb waves is achieved, damage classification is based on the comparison between the original input waveform and the reconstructed signal:

$$DI = 1 - \sqrt{\frac{\left\{ \int_{t_0}^{t_1} I(t)V(t) dt \right\}^2}{\left\{ \int_{t_0}^{t_1} I(t)^2 dt \int_{t_0}^{t_1} V(t)^2 dt \right\}}} \tag{1}$$

where the  $I(t)$  and  $V(t)$  denote the known input and reconstructed signals.  $t_0$  and  $t_1$  represent the starting and ending time points of the baseline signal's first  $A_0$  mode. The value of  $DI$  becomes zero when the time reversibility of Lamb waves is preserved. Note that the root square term in Equation (1) becomes 1.0 if and only if  $V(t) = \beta I(t)$  for all  $t$  where  $t_0 \leq t \leq t_1$  and  $\beta$  is a nonzero constant. Therefore, a simple linear attenuation of a signal will not alter the damage index value. If the reconstructed signal deviates from the input signal, the damage index value increases and approaches 1.0, indicating the existence of damage along the direct wave path.

Once the damage index value exceeds a threshold value, the corresponding signal is defined as damaged in a conventional approach. Here, the critical question is how to set the threshold value so that misclassification of damage can be minimized. The common practice has been to collect the damage index values from the baseline condition of the system and to characterize the distribution of the damage index values. Once the distribution of the damage index value is properly estimated, a threshold value can be established for a user-specified confidence level. A challenge for the proposed method comes from the fact that the decision-making for damage classification needs to be set up without using prior baseline data. To accomplish this task, a consecutive outlier analysis is employed to identify damage without relying on the past baseline data.



(a) A schematic concept of TRA-based damage identification that does not require any past baseline signals

(b) The violation of time reversibility due to damage

Figure 2: A damage detection method based on a time reversal process is developed for online continuous monitoring of in-service structures as an unmanned aerial vehicle predator.

### STATISTICAL DAMAGE CLASSIFICATION

Damage diagnosis is performed by applying an outlier analysis to the damage index values obtained in the previous section. In this section, the basic concept of the outlier analysis is briefly reviewed and extended to the consecutive outlier analysis for detecting multiple outliers. In particular, the outlier analysis is formulated specifically for data from an exponential distribution, and extreme value distribution is introduced for converting any extreme distribution to an exponential distribution. Once wave propagation paths affected by delamination are detected using the consecutive outlier analysis, the location and area of delamination are also identified based on the knowledge of the damaged paths. The algorithms developed for the subsequent damage localization and quantification are described in [16].

The objective of the outlier analysis is to identify a new pattern that differs from previously obtained patterns in some significant respect. The concept of the outlier analysis is not entirely new and applications in other fields can be found in literature [21]. For the current specific application, this concept of the outlier analysis is extended to a case, in which there can be multiple outliers. Because there can be more than one outlier in the experiment described (that is, delamination can affect damage index values corresponding to multiple paths), the possibility of multiple outliers in the data needs to be considered.

The issue of multiple outliers is addressed by employing a consecutive outlier analysis [1]. First, the damage index values are instantaneously computed from a questionable state of the system. Second, the damage index values from all the paths are sorted in an ascending order. Third, the largest damage index value is tested for discordance against the remaining damage index values. Then, this last step is repeated for the second largest outlier, the third and so on until all outliers are identified or a predetermined number of damage index values are tested for discordance. It should be noted that each damage index value is tested for discordance with respect to the other simultaneously obtained damage index values rather than with respect to the damage index values obtained from the baseline condition of the structure. Therefore, the damage index values corresponding to the damaged paths have been identified without referencing to the baseline damage index values. In this way, the

dependence on baseline data has been fully removed both in the feature extraction procedure (using the time reversal process) and in the decision-making procedure (using the consecutive outlier analysis).

Majority of the published work on outlier analysis was in the context of normal distribution. It should be noted that a normal distribution weighs the central portion of data rather than the tails of the distribution. In our particular damage detection application, we are mainly concerned with the maximum values of the damage index, because the outliers corresponding damage will reside near the tails of the distribution. The solution to this problem is to use a statistical tool called extreme value statistics (EVS) [3], which is designed to properly model the behavior of a distribution in the tails. The pivotal theorem of EVS states that in the limit as the number of vector samples tends to infinity, the induced distribution on the maxima of the samples can only take one of three forms: Gumbel, Weibull, or Frechet [8]:

$$\text{Gumbel:} \quad F(x) = \exp\left[-\exp\left(-\frac{x-\lambda}{\delta}\right)\right] \quad -\infty < x < \infty \quad (2)$$

$$\text{Weibull:} \quad F(x) = \begin{cases} 1 & \text{if } x \geq \lambda \\ \exp\left[-\left(-\frac{x-\lambda}{\delta}\right)^\beta\right] & \text{otherwise} \end{cases} \quad (3)$$

$$\text{Frechet:} \quad F(x) = \begin{cases} \exp\left[-\left(\frac{x-\lambda}{\delta}\right)^{-\beta}\right] & \text{if } x \geq \lambda \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where  $\lambda$ ,  $\delta$ , and  $\beta$  are the shape parameters, which should be estimated from the data ( $\delta > 0$ ,  $\beta > 0$ ). In a similar fashion, there are only three types of distribution for the minima of the samples. In this study, the EVS is used to convert any extreme value distribution to an exponential distribution so that the following consecutive outlier analysis can be used.

### CONSECUTIVE OUTLIER ANALYSIS

First, a consecutive outlier analysis is formulated for exponential samples. Then, data transformation is introduced so that the consecutive outlier analysis can be used for extreme value distributions such as Gumbel, Frechet and Weibull. An exponential distribution with a scale parameter  $b$  and an origin at  $a$  has the following probability density distribution:

$$f(x) = \begin{cases} \frac{1}{b} \exp[-(x-a)/b] & \text{for } x > a \\ 0 & \text{for } x < a \end{cases} \quad (5)$$

There are a wide range of discordant (outlier) tests that can be used with exponential data [1]. In this study, one of the most common tests is presented for demonstration rather than exhaustively comparing different types of available tests. An outlier test for the single smallest sample in an exponential sample is first formulated. (Later on, it becomes clear why the outlier test is conducted for the minimum value rather than the maximum value.) A test statistic for the smallest potential outlier is defined as [11]:

$$T = \frac{X_1}{\sum X_i} \quad (6)$$

where samples  $X_1, X_2, \dots, X_n$  are sorted in an ascending order, and  $n$  is the size of the samples. Then, it can be shown that this test statistic has a probability density function  $f_n(t)$ :

$$f_n(t) = \begin{cases} n(n-1)(1-nt)^{n-1} & \text{for } 0 \leq t \leq \frac{1}{n} \\ 0 & \text{for } t \geq \frac{1}{n} \end{cases} \quad (7)$$

The significance probability associated with an observed value  $t$  of a discordance statistic  $T$  is denoted by  $SP(t)$ :

$$SP(t) < nF_{2,2(n-1)}[(n-1)t/(1-t)] \quad (8)$$

where  $F_{v,u}[x]$  is a F-cumulative distribution function with  $v$  and  $u$  degrees of freedom.  $SP(t)$  is the probability that  $T$  takes values more discordant than  $t$ . In another word, the probability that there will be other smaller outliers more discordant than  $t$ . That means when the  $SP(t)$  is small for an observed value of  $t$ , the smallest value  $X_1$  associated with  $t$  is most likely an outlier. Therefore, we define an outlier probability  $OP(t)$  as:

$$OP(t) > 1 - nF_{2,2(n-1)}[(n-1)t/(1-t)] \quad (9)$$

This outlier analysis is consecutively conducted starting from the smallest value to the second smallest value, the third and so on until all outliers are identified or the maximum number of samples specified by a user is reached.

Now, if  $X$  is a sample from the Gumbel maximum distribution, then the following transformed sample  $Y$  has an exponential distribution with origin 0 and mean  $\exp[-\lambda/\delta]$ :

$$Y = \exp\left[-\frac{X}{\delta}\right] \quad (10)$$

Thus, if the parameter  $\delta$  defined in Equation (2) is known ( $\lambda$  does not necessary have to be known), an outlier in the Gumbel samples can be tested by applying to the transformed samples the consecutive outlier analysis for an exponential sample. Note that the largest value  $X_n$  in the original Gumbel sample is converted to the smallest value  $Y_1$  in the transformed sample. Therefore, the test on the  $Y$  values must be chosen accordingly. This is why the consecutive outlier test is previously formulated for the minimum value of the exponential distribution.

Similar transformations exist for the Weibull and Frechet distributions. If the values of  $\lambda$  and  $\beta$  defined in Equations (4) and (3) are known, the following transformations convert a Weibull sample or a Frechet sample into an exponential sample with mean  $\delta^\beta$  or  $\delta^{-\beta}$ , respectively.

$$Y = |X - \lambda|^\beta \text{ or } Y = |X - \lambda|^{-\beta} \quad (11)$$

Once extreme value samples are transformed to exponential samples, the rest of the consecutive outlier analysis is identical to that of the exponential samples.

Note that for the outlier test of a sample  $X_i$ , the best-fit extreme value distribution and the associated  $\delta$ ,  $\lambda$ , and  $\beta$  parameters need to be estimated for the remaining data set  $\{X_1, X_2, \dots, X_{i-1}\}$ . In addition, the procedure is repeated in a consecutive manner for  $i=n, n-1, \dots, n-k$ . Here,  $k$  is the maximum number of possible outliers that will be tested for discordance, and the issue of determining the upper limit of possible outliers has been studied by various researchers [19]. Because the selection of the extreme value distribution and the associated parameter estimation need to be sequentially performed multiple times, algorithms have been developed to automate this procedure [17].

## EXPERIMENTAL RESULTS

In this study, typical results only from one of impact tests are presented due to the space limitation. The active sensing system and the proposed damage identification algorithms were employed to identify internal delamination. Figure 3 (a) shows the actual impact location. The identification of the damaged paths shown in Figure 3 (b) is based on the premise that if there is any defect along the wave propagation path, the time reversibility of Lamb waves breaks down. Therefore, by examining the deviation of the reconstructed signal from the known original input signal for each path damaged paths can be identified. The final goal is to pinpoint the location of delamination and to estimate its size based on the damaged paths identified in Figure 3 (b). To identify the location and area of the delamination, a damage localization algorithm is also developed in [16]. The delamination location and size estimated by the active sensing system was presented in Figure 3 (c), and the estimate from the proposed damage identification matched well with the result of ultrasonic scan.

Based on the time reversibility, the damage index defined in Equation (1) was first computed for all 66 paths when there was no delamination on the plate. Following the consecutive outlier analysis previously described, the damage index values after the impact were first arranged in an ascending order as shown in Figure 4 (a). Then, the outlier probability defined in Equation (9) is computed in Figure 4 (b) for the first 15 largest damage index values. Here, the maximum possible number of outliers is set to 15 ( $k=15$ ) based on the assumption that damage is localized and only a few wave propagation paths are affected by the delamination.

The outlier probabilities for the first 15 largest damage index values in Figure 4 (b) also are listed in Table 1. The outlier probability for the largest damage index values is about 49.4%, and the outlier probability reaches its maximum value at the fifth largest damage index value ( $OP(t) = 0.99999900891873$  for the fifth largest damage index value). For a given confident level of 99.9%, the fifth largest damage index value is classified to be an outlier, and this automatically implies that the larger values damage index values  $X_{63}$ ,  $X_{64}$ ,  $X_{65}$ , and  $X_{66}$  are outliers

as well. It should be noted that the outlier probabilities for the first three largest damage index values were relatively low due to the masking effect. The masking effect is the insensitivity of an outlier analysis to identify an outlier in the presence of several suspected outliers closer to each other than to the bulk of the remaining observations [22]. For example, with the damage index values shown in Table 2, the outlier probability of  $X_n=0.9147$  was low because of the proximity of  $X_{n-1}=0.9120$ ,  $X_{n-2}=0.9013$ ,  $X_{n-3}=0.8446$ ,  $X_{n-4}=0.8054$ . Here  $X_{n-1}$ ,  $X_{n-2}$ , ...,  $X_{n-4}$  are said to have a masking-effect on the identification of  $X_n$ . Similar masking effects were observed for  $X_{n-1}$ ,  $X_{n-2}$ , ...,  $X_{n-4}$ . However, the masking effect were alleviated as the consecutive outlier analysis moved inward, and it was fully removed at  $X_{62=n-5}$ .

Once the number of actual outliers was identified, the automated procedure for the selection of the best-fit extreme distribution and parameter estimation was applied to the remaining damage index values  $X_i$  ( $i=1, 2, \dots, n-6$ ). The Weibull distribution was chosen as the best-fit extreme distribution, and the associated  $\lambda$ ,  $\beta$ , and  $\delta$  parameters were estimated to be 0.627, 14.992, and 0.596, respectively. From the estimated Weibull distribution, a threshold corresponding to a 99.9% confidence level turned out to be 0.251 as shown in Figure 4 (d). It should be noted that the correct outliers were already identified using the consecutive outlier analysis described in Figure 4 (b), and this additional step of establishing the threshold value in Figure 4 (c) and (d) was simply taken to substantiate the findings of the consecutive outlier analysis.

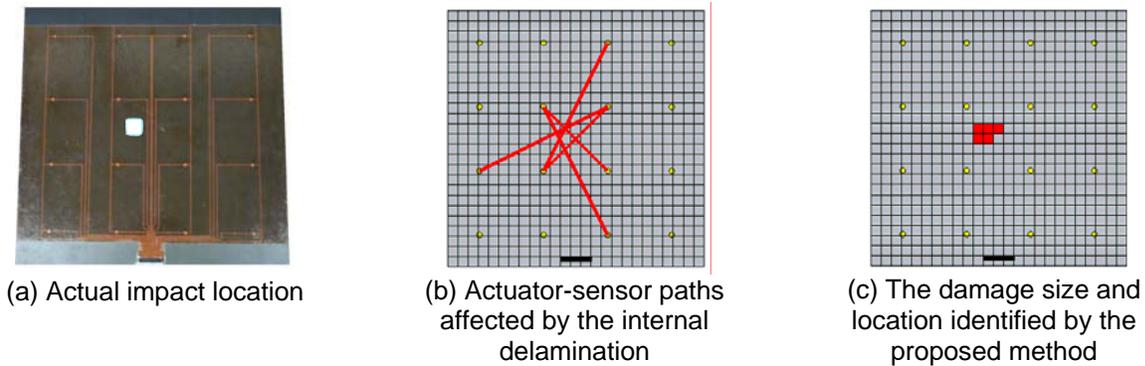


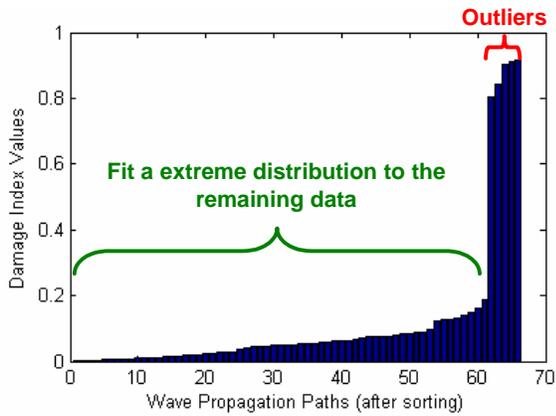
Figure 3: Damage localization and quantification based on a time reversal process

## CONCLUSION

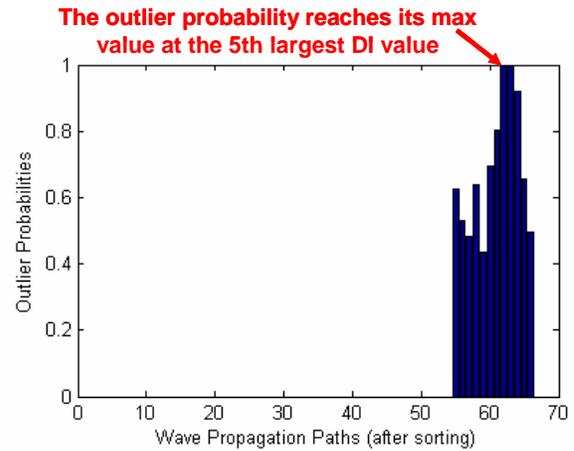
In this study, a combination of a time reversal process and a consecutive outlier analysis is adopted to identify delamination in a composite plate without relying on prior baseline data. Surface mounted PZT materials are used to apply local excitations to the composite plate and to measure dynamic strain time response signals. Then, a damage-sensitive feature is extracted based on the time reversal concept, and a statistical damage classifier is developed via the consecutive outlier analysis. The effectiveness of the proposed method is demonstrated using experimental data obtained from impact tests of composite plates, and the location and area of the delamination is successfully identified.

## ACKNOWLEDGMENT

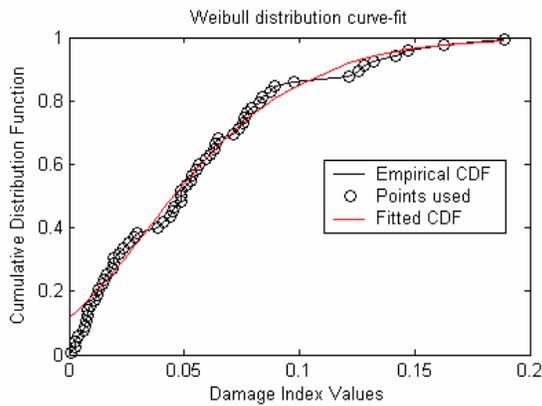
The experiment was performed at Los Alamos National Laboratory and funded by the Department of Energy through the internal funding program at Los Alamos National Laboratory known as Laboratory Directed Research and Development (Damage Prognosis Solutions).



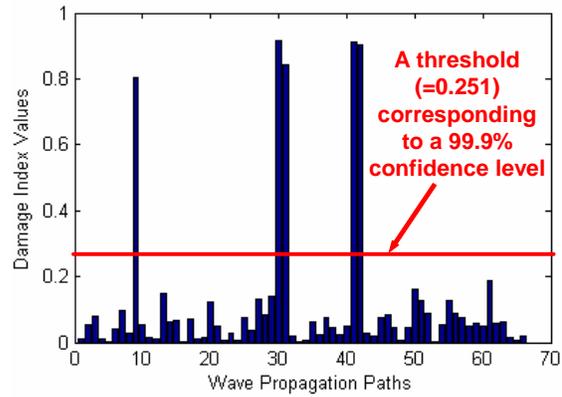
(a) The DI values are sorted in an ascending order and the damaged paths (outliers) are identified from the outlier probability in Figure 4 (b)



(b) The outlier probability is computed for the first 15 largest damage index values, and it reaches its maximum value at the 5th largest damage index value



(c) After excluding the 5 outliers identified in Figure 4 (b), a Weibull distribution is fitted to the remaining 63 DI values to estimate a new threshold value (=0.251).



(d) The use of the threshold value (0.251) confirms that the 5 largest DI values are outliers and the associated paths are influenced by internal delamination

Figure 4: Threshold establishment and instantaneous damage identification without using prior baseline data

**Table 1: The outlier probabilities for the first 12 largest damage index values shown in Figure 4 (b)**

#	Outlier Probability	#	Outlier Probability	#	Outlier Probability
1	0.49443100201884	5	<b>0.99999900891873</b>	9	0.63736975495153
2	0.65388841395516	6	0.80118097717756	10	0.48095139238444
3	0.92056208261950	7	0.69331542635726	11	0.52869657681871
4	0.99960175642805	8	0.43415729821632	12	0.62566554445656

**Table 2: The first 12 largest damage index values shown in Figure 4 (a)**

#	Damage Index Values	#	Damage Index Values	#	Damage Index Values
1	0.9147	5	0.8054	9	0.1419
2	0.9120	6	0.1888	10	0.1319
3	0.9013	7	0.1628	11	0.1280
4	0.8446	8	0.1470	12	0.1258

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